Optimal local LPV identification experiment design

D. Ghosh[(1)], X. Bombois[(1)], J. Huillery[(1)], G. Scorletti[(1)] et G. Mercère[(2)]

1. Laboratoire Ampère UMR CNRS 5005
2. LIAS, Université de Poitiers

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Introduction

An LPV system is a system whose parameters depend on an exogenous (scheduling) variable $p(t)$

If $p(t)$ is kept constant , the LPV system is an LTI system

The dynamics of this LTI system depend on the value of the constant p

We have a collection of LTI dynamics at different operating points

Such a representation can be used to deal with non-linear systems (gain scheduling)

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Local LPV identification approach: $p(t)$ is kept constant at successive operating points and local LTI identification experiments are performed

We determine those operating points and the local LTI identification experiments to guarantee a certain model accuracy with the least input energy

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Description of the LPV system

We consider the following LPV-OE system for simplicity:

$$y_{nf}(t) = -\sum_{i=1}^{n_a} a_i^0(p(t))\ y(t-i) + \sum_{i=1}^{n_b} b_i^0(p(t))\ u(t-i)$$

$$y(t) = y_{nf}(t) + e(t)$$

The parameter vector $\xi^0(p(t)) = (a_1^0(p(t)), ..., b_{n_b}(p(t)))^T$ depends on the time-varying scheduling variable $p(t)$

$$a_i^0(p(t)) = a_{i,0}^0\ +\ \sum_{j=1}^{n_p} a_{i,j}^0\ p^j(t)$$

$$b_i^0(p(t)) = b_{i,0}^0\ +\ \sum_{j=1}^{n_p} b_{i,j}^0\ p^j(t)$$

Description of the LPV system

$$a_i^0(p(t)) = a_{i,0}^0 \; + \; \sum_{j=1}^{n_p} a_{i,j}^0 \; p^j(t) \quad i = 1...n_a$$

$$b_i^0(p(t)) = b_{i,0}^0 \; + \; \sum_{j=1}^{n_p} b_{i,j}^0 \; p^j(t) \quad i = 1...n_b$$

This defines a mapping $T(p(t))$ between the global parameter vector θ^0 and the time-varying parameter vector $\xi^0(p(t))$

$$\xi^0(p(t)) = T(p(t)) \; \theta^0$$

$$\xi^0(p(t)) = (a_1^0(p(t)), ..., b_{n_b}(p(t)))^T \quad \theta^0 = (a_{1,0}^0, ..., b_{n_b,n_p}^0)^T$$

Identification objective

$$\xi^0(p(t)) = T(p(t))\ \theta^0$$

The parameter vector θ^0 entirely determines the LPV system

Objective. Determine with the least powerful excitation an estimate $\hat{\theta}$ of θ^0 having a given accuracy:

$$P_\theta^{-1} > R_{adm}$$

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Identification of an LPV system: local approach

Suppose $p(t)$ is kept constant to an operating point $\mathbf{p}_m$

$$p(t) = \mathbf{p}_m \ \ \forall t$$

The LPV system then reduces to an LTI system described by a time-invariant parameter vector $\xi^0(\mathbf{p}_m)$

$$y(t) = G(z, \xi^0(\mathbf{p}_m))u(t) + e(t)$$

$$\xi^0(\mathbf{p}_m) = T(\mathbf{p}_m)\ \theta^0$$

This LTI system can of course then be identified using LTI prediction error identification

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LTI identification at an operating point $\mathbf{p}_m$

If we apply an input signal u_m of spectrum Φ_{u_m} to

$$y_m(t) = G(z, \xi^0(\mathbf{p}_m))u_m(t) + e_m(t),$$

we can collect a data set $Z_m^N = \{y_m(t),\ u_m(t) \mid t = 1...N\}$ and identify an estimate $\hat{\xi}_m$ of $\xi^0(\mathbf{p}_m)$ using:

$$\hat{\xi}_m = \arg\ \min_{\xi} \frac{1}{N} \sum_{t=1}^{N} (y_m(t) - G(z, \xi)u_m(t))^2$$

This estimate is (asymptotically) such that $\hat{\xi}_m \sim \mathcal{N}(\xi^0(\mathbf{p}_m), P_{\hat{\xi}_m})$

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The estimate $\hat{\xi}_m$ is (asymptotically) such that $\hat{\xi}_m \sim \mathcal{N}(\xi^0(\mathbf{p}_m), P_{\hat{\xi}_m})$

The covariance matrix $P_{\hat{\xi}_m}$ depends on $\xi^0(\mathbf{p}_m)$ and Φ_{u_m}:

$$P_{\hat{\xi}_m}^{-1} = \frac{N}{\sigma_e^2} \frac{1}{2\pi} \int_{-\pi}^{\pi} F(e^{j\omega}, \xi^0(\mathbf{p}_m)) \; F(e^{j\omega}, \xi^0(\mathbf{p}_m))^* \; \Phi_{u_m}(\omega) d\omega$$

$$F(z, \xi^0(\mathbf{p}_m)) = \left. \frac{dG(z,\xi)}{d\xi} \right|_{\xi^0(\mathbf{p}_m)}$$

This operation has to be repeated at different $\mathbf{p}_m$ to deduce an estimate of θ^0 since $dim(\xi^0(\mathbf{p}_m)) < dim(\theta^0)$

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We obtain M estimates $\hat{\xi}_m$ of $\xi^0(\mathbf{p}_m) = T(\mathbf{p}_m)\ \theta^0$:

$$\hat{\xi}_m = T(\mathbf{p}_m)\ \theta^0 + \delta_m \qquad \delta_m \sim \mathcal{N}(0, P_{\hat{\xi}_m})$$

The estimate $\hat{\theta}$ of θ^0 is classically determined using ordinary least squares based on the observations $\hat{\xi}_m$ and the regressor $T(\mathbf{p}_m)$

This is however not the minimum variance estimator since the respective variances of $\hat{\xi}_m$ are neglected

$\Longrightarrow$ use of weighted least squares:

$$\hat{\theta} = \arg\min_{\theta} \sum_{m=1}^{M} \left(\hat{\xi}_m - T(\mathbf{p}_m)\theta\right)^T P_{\hat{\xi}_m}^{-1} \left(\hat{\xi}_m - T(\mathbf{p}_m)\theta\right)$$

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The estimate $\hat{\theta}$ is such that $\hat{\theta} \sim \mathcal{N}(\theta^0, P_\theta)$

$$P_\theta^{-1} = \sum_{m=1}^{M} T^T(\mathbf{p}_m)\ P_{\hat{\xi}_m}^{-1}\ T(\mathbf{p}_m)$$

with $P_{\hat{\xi}_m}^{-1}$ linear in Φ_{u_m}

P_θ^{-1} is the sum of the contribution of each local experiments !!

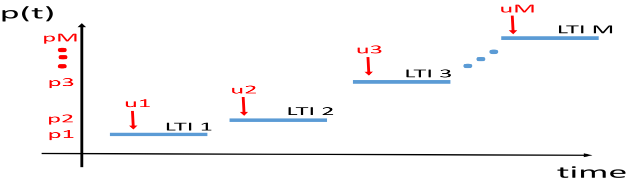
Optimal experimental design

To-be-optimized variables:

- the number M of local identification experiments M,
- the operating points $\mathbf{p}_m$ $(m = 1...M)$
- the spectra Φ_{u_m} of the input signal u_m $(m = 1...M)$ used in the local identification experiments

To-be-minimized cost: $\mathcal{J} = N \sum_{m=1}^{M} \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{u_m}(\omega)\, d\omega$

Accuracy constraint: $P_\theta^{-1} > R_{adm}$



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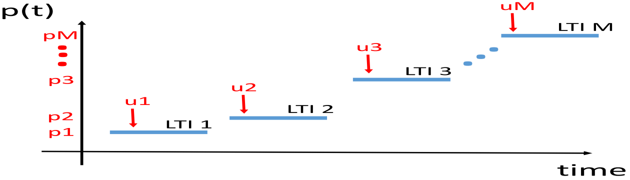

Convex optimization for the design of the spectra

Suppose that we have a-priori chosen M and $\mathbf{p}_m$ $(m = 1...M)$

The design of Φ_{u_m} $(m = 1...M)$ is then a convex optimization problem

$$\min_{\Phi_{u_m}\ (m=1...M)} N \sum_{m=1}^{M} \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{u_m}(\omega)\, d\omega$$

$$\sum_{m=1}^{M} T^T(\mathbf{p}_m)\ P_{\hat{\xi}_m}^{-1}(\Phi_{u_m})\ T(\mathbf{p}_m) > R_{adm}$$

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How to perform the selection of the operating points $\mathbf{p}_m$?

$$\min_{\Phi_{u_m}} N \sum_{m=1}^{M} \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{u_m}(\omega)\, d\omega$$
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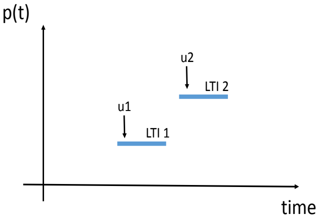

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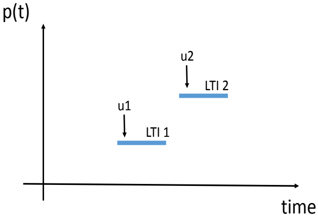

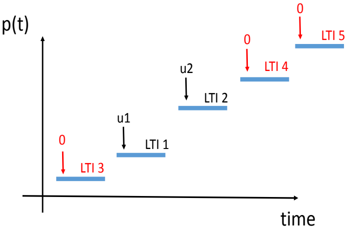

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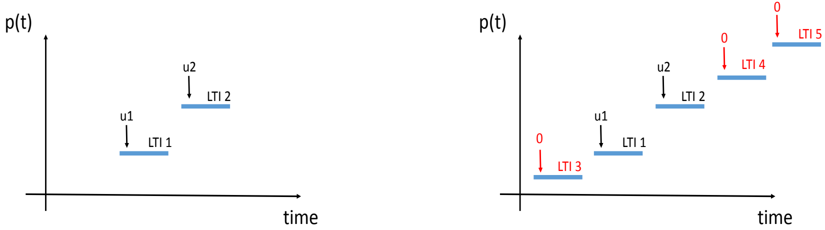

These experiments are equivalent from a mathematical point of view since they lead to the same cost $\mathcal{J}$ and the same P_θ^{-1} !!

Convex formulation of the experiment design problem

Consider a fine grid $\{\mathbf{p}_1,\ \mathbf{p}_2, ..., \mathbf{p}_{M_{grid}}\}$ of the scheduling space

We will determine a spectrum Φ_{u_m} for all $\mathbf{p}_m$ in this fine grid

The optimal experiment design problem can thus be formulated as:

$$\min_{\Phi_{u_m}\ (m=1...M_{grid})} N \sum_{m=1}^{M_{grid}} \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{u_m}(\omega)\ d\omega$$

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Chicken-and-egg problem

The covariance matrix P_θ depends on θ^0

We can determine a first estimate θ_{init} of θ^0 using an initial local LPV identification

The optimal experiment design problem will then be used to complement the information delivered by this initial experiment

In this optimal experiment design problem, θ^0 will be replaced by θ_{init}

Numerical illustration

Consider the following LPV-OE system: $y(t) = y_{nf}(t) + e(t)$

$$y_{nf}(t) = -a_1^0(p(t))\ y(t-1) + \ b_1^0(p(t))\ u(t-1)$$

$$a_1^0(p(t)) = -0.9\ +\ 0.1\ p(t) \qquad b_1^0(p(t)) = 10\ -\ 1\ p(t)$$

$$\underbrace{\begin{pmatrix} a_1^0(p(t)) \\ b_1^0(p(t)) \end{pmatrix}}_{=\xi^0(p(t))} = \underbrace{\begin{pmatrix} 1 & p(t) & 0 & 0 \\ 0 & 0 & 1 & p(t) \end{pmatrix}}_{=T(p(t))} \underbrace{\begin{pmatrix} -0.9 \\ 0.1 \\ 10 \\ -1 \end{pmatrix}}_{=\theta^0}$$

$p(t)$ can take values in the scheduling space [0 8]

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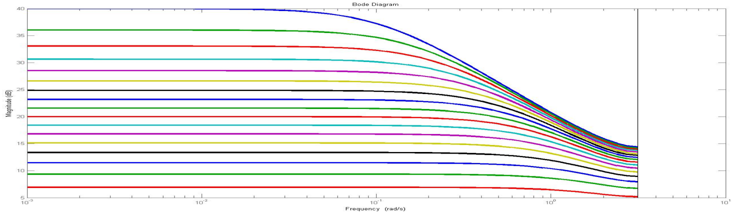
Frequency responses of the corresponding $G(z, \xi^0(\mathbf{p}_m))$



We choose: $N = 1000$, $\sigma_e^2 = 0.5$ and R_{adm} enforces a standard deviation of 0.3% on each parameter of θ^0

Optimization problem based on the $M_{grid} = 17$ operating points

$$\mathbf{p}_m = 0,\ 0.5,\ 1,\ 1.5, \ldots,\ 8$$

$\Longrightarrow$ only three nonzero Φ_{u_m} at $\mathbf{p}_m = 0,\ 1$ and 8

Corresponding $G(z, \xi^0(\mathbf{p}_m))$ and Φ_{u_m}

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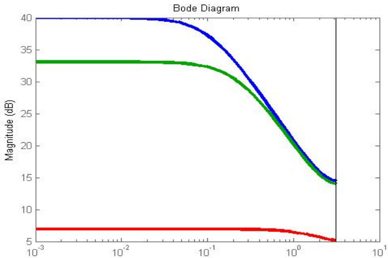

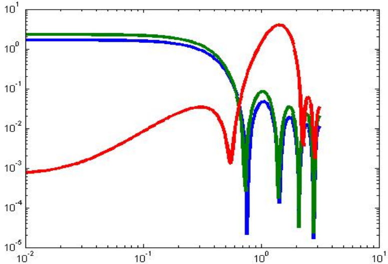

Let us compare the required input energy $\mathcal{J}$ to obtain $P_\theta^{-1} > R_{adm}$ for different choices of $\mathbf{p}_m$

$\mathbf{p}_m$	required input energy $\mathcal{J}$
$\mathbf{p}_m = 0,\ 1,\ 8$	1380
$\mathbf{p}_m = 0,\ 4,\ 8$	2320
$\mathbf{p}_m = 0,\ 1$	23000
$\mathbf{p}_m = 0,\ 8$	16000
$\mathbf{p}_m = 1,\ 8$	23000

Conclusions

First attempt to tackle the optimal experiment design problem for LPV systems

Local approach: $p(t)$ follows a staircase shape

A staircase $p(t)$ is certainly not (fully) optimal

Future work will consider other shapes of $p(t)$ (global LPV identification)

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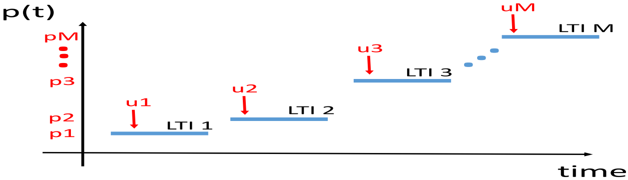

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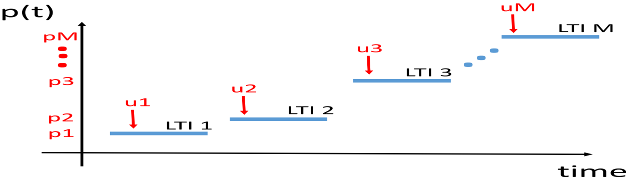

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